Reconstruction of Parallel and Fan Beam Projections for Biomedical Diagnosis

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Abstract—Computed Tomography (CT) is a medical imaging technique which produces cross sectional images of human tissues representing their x-ray attenuation coefficients. The attenuation coefficients are recorded as projections for a given cross sectional slice of the body. Reconstruction of those projections yields the 2D image of the required slice. Depending on the mode of acquisition of projections, geometry of X-ray source and detectors, arrangement of detectors, there are many types of CT. The reconstruction algorithm for each type is different. In this paper, we created a set of phantom (cross sections of thoracic region), and acquired projections for them. Both conventional and spiral projections are acquired. Reconstruction of these projections is done and the final cross sectional images are compared for conventional CT and spiral CT. Matlab R2009a is used for simulation of phantoms, acquisition of projections and reconstruction of it and Image 1.44 is used for quantitative analysis.

Index Terms—Computed Tomography (CT), Conventional CT, Spiral CT, Phantom, Matlab, Image J

I. INTRODUCTION

Biomedical Engineering is integrated biology and medicine with engineering field. It combines design and development of new medicines and instruments for the healthcare purposes [1]. The history of CT began in 1985 when Wilhelm Roentgen discovered x-rays [2]. In X-ray medical imaging, the attenuation coefficients of human tissues are recorded in photographic plate. It provides projections for only one direction. The subject is made to lie on the movable bed in appropriate position for imaging a particular section of body. The gantry has the x-ray source and detectors and they are located opposite to each other [3] X-rays are made to pass through the section of the subject and the attenuation coefficients of the tissues in that section are recorded in the form of projections by the detectors placed on the other side.

More projections of the section are obtained by rotating the x-ray source and detector around the stationary subject. After completing one rotation around the subject, the bed moves inside the gantry and the projections are recorded for the next section or slice of the subject (Conventional CT) [4].

Reconstruction of a function from its projections was first formulated by Johann Radon in 1917 [5]. The first CT scanning machine was developed in 1972 by Hounsfield and Cormack. Before the invention of CT other kinds of tomography like linear tomography, axial transverse tomography existed. Spiral and Multi-slice CT were introduced in 1989 and 1998 respectively. A lot of developments have been achieved in arrangement of source and detectors. Many projection geometries like fan beam, cone beam, parallel beam were developed and used.

A. Advantages and Disadvantages of Spiral CT over Conventional CT

In spiral CT the time used for acquisition of projections is less when compared with conventional CT.

In spiral CT the projections are not collected on a slice by slice basis. In Spiral CT the fundamental requirements of CT are violated. Successive projections are not taken from a same slice and the projections are not self consistent. Virtual projections are constructed by suitable interpolation from adjacent projections [2].

Spiral CT is the preferred mode by the doctors for modern day diagnosis. Sequential slices (conventional CT) are done only when minute details are required. Conventional CT is also known as High Resolution CT.
II. MATERIALS AND METHODS

A. Creation of Phantom Set

We have selected thoracic region and a set of phantom is created for it. 360 phantoms are created. Each phantom is designed to represent a cross section of thoracic region. The changes in the dimensions and shape of lungs and other associated organs and structures (vertebral column, ribs) are taken into consideration while creating phantoms [7]. The changes in the attenuation coefficients of different tissues present in thoracic region are analyzed from different CT images and those changes are considered for allotting gray levels for different structures in the phantom.

B. Reasons for Using Phantom Set

The projection in real time CT data acquisition occupies a huge memory space and it is stored temporarily. It is only accessible to developers and engineers.

C. Parallel Beam Geometry

The phantom set (360) is divided into two equal sets (180 each). Parallel beam geometry and fan beam geometry are used for data acquisition. In parallel-beam geometry, each projection is formed by combining a set of line integrals through an image at a specific angle. Parallel rays of x-rays are made to fall on the subject and the attenuation profiles are recorded as projections. Both the source and the detectors are linear arrays. The distance between the source and the detector doesn’t play a major role in data acquisition in parallel beam geometry [8].

Radon Transform is used for acquiring projections from these 2 phantom sets.

\[ p(r, \theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(r \cos \theta - s \sin \theta, r \sin \theta + s \cos \theta) ds \]

where f(x, y) is the phantom, p(r, \( \theta \)) is the projection of f (x, y). Radon Transform is taken for all 360 phantoms. Radon transform is the projection of image intensity along a radial line oriented at a specific angle.

D. Fan Beam Geometry

In fan beam geometry point source of x-rays is used. The detectors may be linearly arranged or arranged in an arc. The distance between the centres of the object being scanned and the point source of x-rays should be at optimal value so that the angle of the fan beam emanating from the point source covers the entire object. Shown in Fig. 1 (a) & (b)

E. Acquisition of Projection Data from Phantom Set

The first phantom is used for acquiring projection for the angle (0º - 1º). Similarly each phantom is used to acquire projection for corresponding angular positions. p (r, \( \theta \)) can be measured for \( \theta \) ranging to 2\( \pi \). Beams coming from opposite sides yields identical measurement of attenuation profiles resulting in redundant information. Therefore, for parallel beam geometry p (r, \( \theta \)) is measured for \( \theta \) ranging from 0 to \( \pi \). In fan beam geometry the projections are measured for every 2º increment in angle thus using each one from the set of 180 phantoms for each increment.

F. Interpolation

Data are acquired for discrete axial positions (\( z_1 \), \( z_2 \)) and for angular tube positions \( \beta \) ranging from 0 to \( \pi \). In spiral CT data are acquired while \( \beta \) and Z increase continuously. Weighted Interpolation is used to obtain a complete dataset at one particular axial position \( z_1 \). [9]. We reconstructed a slice at a particular angular position \( z_i \) for both circular and spiral CT. In circular CT data for \( \beta \) ranging from 0 to \( \pi \) is available at \( z_i \) position. In the case of spiral CT only 2 views are available at \( z_i \). The remaining value of \( \beta \) at \( z_i \) is obtained by weighted interpolation from measurements at adjacent axial positions. In this paper weights allotted for interpolation from axial measurement depends upon the distance of the axial points from \( z_i \). Lagrange interpolation is used in this work for interpolating the spiral projections before performing reconstruction. Using interpolation the angular range is reduced from 360º to 180º [10].

Weight Factor = 0.00555q

where q increases from 1 to 180 linearly as axial position varies. It is 180 at \( z_i \). It decreases linearly from 180 to 1 after \( z_i \). The projections which are nearer to the point of reconstruction are given more weight age. Linear interpolation is carried out using the following formula.
where \( x=180 \) (the point to which the values are interpolated). \( x_0 \) and \( x_1 \) are axial points (\( z \)) for which we have projection data (\( \beta \)). \( x_0 \) corresponds to data points of first 180 phantoms. \( x_1 \) corresponds to dataset of second 180 phantoms. \( f(x_0) \) and \( f(x_1) \) are the projection values for \( x_0 \) and \( x_1 \). Interpolation is done for projection data acquired using parallel and fan beam geometry.

### III. RESULTS

#### A. Reconstruction

The reconstruction algorithm is same for both circular and spiral CT. In circular CT reconstruction is done directly from projections. In spiral CT the projections are subjected to weighted interpolation and then reconstruction is performed. Iradon function (Inverse Radon Transform) is used to reconstruct circular and spiral CT projections acquired using parallel beam geometry. The projections acquired using fan beam geometry is converted into parallel beam projections and then inverse radon transform is used for reconstruction.

\[
f(x,y) = R^{-1}\{ p(r,\theta) \}
\]

Iradon function uses filtered back projection algorithm to perform inverse radon transform. The filter is designed directly in the frequency domain and then multiplied by the FFT of the projections.

\[
f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(k, \theta) e^{2\pi i kr} dr d\theta
\]

#### B. Quantitative Analysis

The original phantom slice and the reconstructed outputs are analyzed quantitatively using ImageJ. The profile of the image is obtained along a fixed line running through the middle of the image the image as shown in the Fig. 5 (a) & (b). The profile of each image is shown in Fig. 6 (a) & (b). Specified regions of interest (ROIs) – bone and soft tissue are selected and the intensity values in that region are measured for calculating contrast of the image slices in those regions Fig. 8(a) & (b). Table I shows the calculation of contrast for region of interest.

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**Figure 2.** Original phantom slice (180\(^{th}\) phantom of the set)

**Figure 3.** (a) Reconstructed image of slice - for circular CT; (b) spiral CT for parallel beam geometry

**Figure 4.** (a) Reconstructed image - for circular CT; (b) Spiral CT for fan beam geometry

**Figure 5.** (a) Positioning of line for obtaining profile; (b) Profile information of original phantom slice

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**Figure 6.** (a) & (b) Profile information of reconstructed images

**Figure 7.** Gray value vs. Distance (pixels)

**Figure 8.** Gray value vs. Distance (pixels)
TABLE I. CALCULATION OF CONTRAST FOR ROIs OF ORIGINAL PHANTOM AND DIFFERENT OUTPUTS

<table>
<thead>
<tr>
<th>SI NO</th>
<th>Type of image Original Phantom</th>
<th>S (bone)</th>
<th>b (tissue)</th>
<th>Contrast (s-b)/(s+b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Reconstructed image - circular CT – parallel beam geometry</td>
<td>255</td>
<td>177.452</td>
<td>0.1792</td>
</tr>
<tr>
<td>2.</td>
<td>Reconstructed image - spiral CT – parallel beam geometry</td>
<td>255</td>
<td>111.899</td>
<td>0.3900</td>
</tr>
<tr>
<td>3.</td>
<td>Reconstructed image - circular CT – fan beam geometry</td>
<td>255</td>
<td>177.632</td>
<td>0.1788</td>
</tr>
<tr>
<td>4.</td>
<td>Reconstructed image - spiral CT – fan beam geometry</td>
<td>252.773</td>
<td>123.838</td>
<td>0.3423</td>
</tr>
</tbody>
</table>

IV. DISCUSSION

In the Iterative approach (ART–Algebraic Reconstruction algorithm) was used earlier for reconstruction. Based on errors between projections that would be obtained from current pixel values and the actual projections, each pixel was updated in this algorithm. In this paper, algorithm is same for both Spiral and circular CT, in the spiral CT the weighted interpolation has performed for reconstruction from projections. But in the circular CT it can be done directly from projections. Based on this algorithm the errors are rectified between the projections from current pixel value and each pixel is updated in the algorithm.

V. CONCLUSION

In this paper, spiral projections are obtained from phantom set using parallel and fan beam geometry and it is reconstructed after performing weighted interpolation on the projections. Radon Transform is used for acquiring projections and Inverse Radon Transform is used for reconstructing from projections. Reconstructed images are compared. Contrast is good in spiral CT when compared to circular CT.

ACKNOWLEDGEMENT

Authors deliver their gratitude to FIST-DST Program (No. SR/FST/College-189/2013) Govt. of India for laboratory facilities support.

REFERENCES


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