Skin Prestretch Evaluation Adapting Mooney-Rivlin Model

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***Abstract*—This paper aims to assess the influence of skin prestretch to the stress-stretch relations in defining skin behaviour and properties via a parametric study. Skin prestretch evaluation based on the mathematical model derivation of Mooney-Rivlin constitutive equation is presented in the form of stress-stretch curves. Prestretch term, λ_p, is included in the re-derivation of engineering stretch, σ_E and principal stretch, λ, relations. A set of skin prestretch parametric study along with a Matlab programme are designed and executed. The results show that prestretch terms influence the stress-stretch curve behaviour along with other Mooney-Rivlin material constants (C_1 and C_2). It was found that 0.2 unit difference in C_1 (ΔC_1) produced the largest effect compared to 0.2 unit difference in C_2 (ΔC_2) and prestretch λ_p ($\Delta\lambda_p$). The knowledge and information related to hyperelastic constitutive model gained from this study is beneficial to the development of skin constitutive model.**

***Index Terms*—mooney-rivlin model, hyperelastic material, skin prestretch, stress-stretch curve**

I. Introduction

The investigation of complex skin properties has been one of the main interests in skin study. This is believed due to the importance of synthetic skin development and its significance in many applications. High demand for skin replacements or skin substitutes mainly for medical and cosmetics purpose [1] therefore had enhanced in-depth study in producing an ultimate skin replacement which is identical to the real skin properties biologically or mechanically. But unfortunately to date, this is not yet being achieved [2]. Even so, continuous research will contribute to better understanding of human skin behaviour and properties as well as to the synthetic skin development in general.

In quantifying skin properties and behaviour, experimental techniques have been widely employed compared to numerical or computational technique [3]-[6]. At current, no numerical model with its constitutive equation explicitly for human skin has been recognized. The theory of linear elasticity (e.g. Hooke's law) is insufficient in representing materials with highly nonlinear behaviour and therefore, nonlinear elastic theory was developed [7]. Groves, Mahmud *et al.*, and Wang *et al.* [8]-[10] mentioned that the adaptation of hyperelastic model in describing hyperelastic material (e.g. skin, soft tissues and rubber-like material) was started since the 70's, disregarded its viscoelasticity characteristic. Some of commonly employed hyperelastic constitutive model are Neo-Hookean [7], [8], [11], [12] Mooney-Rivlin [7], [8], [11], [13]-[15] Ogden [3], [7], [8], [9], [11], [14], [15] Veronda-Westmann [7], [8], Yeoh [7], [8], [15] Arruda-Boyce [8], [16] and etc.

Generally skin shall be treated as hyperelastic material [17]-[19] because to model a multilayered, anisotropic, viscoelastic skin would be very arduous [3]. Therefore, adapting hyperelastic constitutive model in determining skin behaviour and properties seems to be a convenient resolution. Each model has their very own strain energy density functions, W formulated to represent the materials behaviour in a way of stresses to strains relation.

Assumed to hold isotropic and almost incompressible characteristic as indicated in Rivlin's phenomenological theory [20], general strain energy density functions, W of Mooney-Rivlin material model for soft materials (i.e. human skin), may be written as in Equation (1) as follows [11]:

$$W = C_1\left(\overline{I}_1 - 3\right) + C_2\left(\overline{I}_2 - 3\right) \tag{1}$$

where material constants are C_1 and C_2; meanwhile $\bar{I}_1$ and $\bar{I}_2$ are the first and second invariant of left Cauchy-Green deformation tensor's element.

Thus on account of limited study through numerical assessment in the current literature, this paper for the first time investigates the effects of skin pre-stretch based on Mooney-Rivlin constitutive equation via numerical approach. A parametric study on skin prestretch adopting Mooney-Rivlin model is novel due to no similar technique has been reported previously.

II. Methodology

In evaluating the influences of skin prestretch to the stress-stretch relation in determining skin behaviour and properties, investigation stages were systematically designed and conducted. Fig. 1 illustrates the skin prestretch investigation process flow.

A. Stage 1: Mooney-Rivlin Material Model Disregard Prestretch Terms

In the first stage of the investigation, general Mooney-Rivlin constitutive equation was derived to highlight the

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relation between engineering stress, σ_E and principal stretch, λ by considering skin to be hyperelastic, incompressible, isotropy and nonlinear material. Disregard prestretch terms, stress-stretch relations obtained as in equation (2) below with material constants C_1 and C_2.

$$\sigma_E = \frac{1}{\lambda}\left(2C_1 + \frac{2C_2}{\lambda}\right) + \left(\lambda^2 - \frac{1}{\lambda}\right) \tag{2}$$

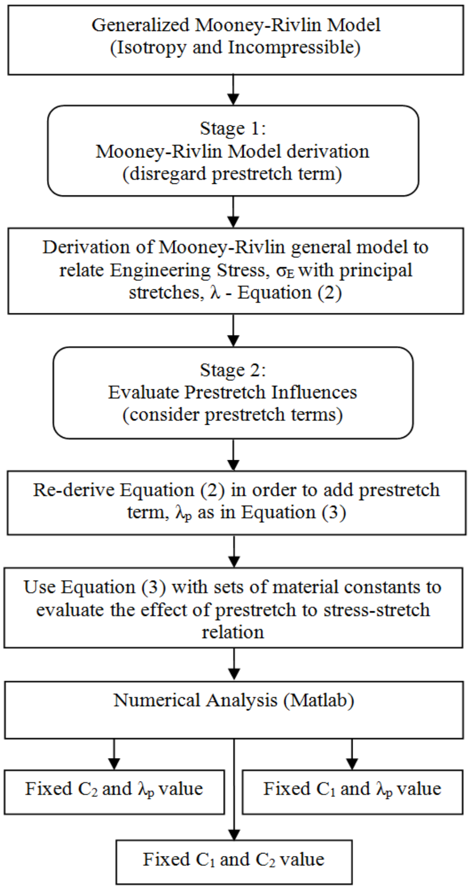

Figure 1. Prestretch evaluation process flow

B. *Stage 2: Mooney-Rivlin Model Evaluation Include Prestretch Terms*

Based on Mooney-Rivlin material model, attempts to evaluate skin prestretch influences in stress-stretch relation was done by re-deriving equation (2) for engineering stress, σ_E and principal stretch, λ by adding the prestretch, λ_p terms as shown in equation (3).

$$\sigma_E = \frac{1}{(\lambda-\lambda_p)}\left(2C_1 + \frac{2C_2}{(\lambda-\lambda_p)}\right) + \left((\lambda-\lambda_p)^2 - \frac{1}{(\lambda-\lambda_p)}\right) \tag{3}$$

By using equation (3), three different sets of material constants were employed as follows:

- Set I: Variation of C_1 value with fixed C_2 and λ_p value.
- Set II: Variation of C_2 value with fixed C_1 and λ_p value.
- Set III: Variation of λ_p value with fixed C_1 and C_2 value.

This skin prestretch parametric study was proposed to analyse the sensitivity of Mooney-Rivlin material model constants which includes prestretch terms in the stress-stretch relation equation. Shergold *et al.* [14] constants (C_1=0.3MPa and C_2=0MPa) is included as a reference meanwhile reference value for prestretch, λ_p is 0.2 which is taken from Evans and Holt [21].

III. Results and Discussion

A. *Stage 1: Mooney-Rivlin Material Model Disregard Prestretch Terms*

As presented in Fig. 1, Fig. 2 and Fig. 3, a reference curve of Shergold *et al.* [14] valued 0.3MPa for C_1 and 0MPa for C_2 was plotted in each set by omitting its prestretch term. Also the second reference curve plotted in each set, a combination of Shergold *et al.* [14] and Evans and Holt [21] which includes prestretch term (C_1=0.3MPa, C_2=0MPa and λ_p=0.2).

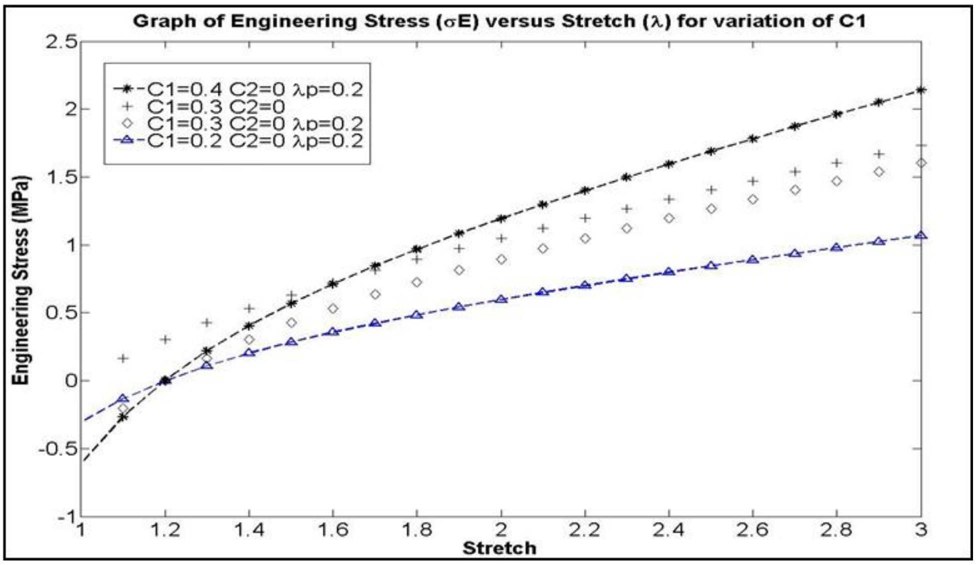

Figure 2. Variations in C1 with fixed C2 and fixed λ_p

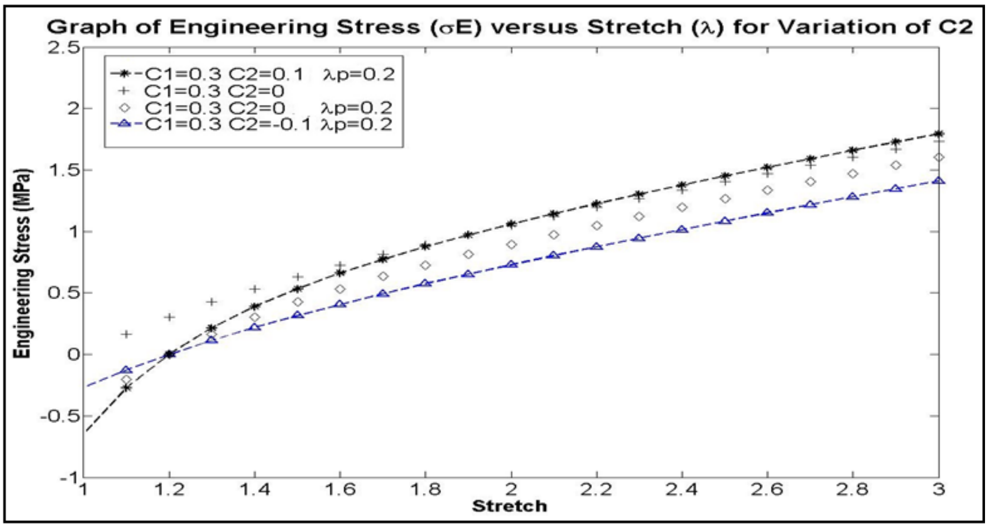

Figure 3. Variations in C2 with fixed C1 and fixed λ_p

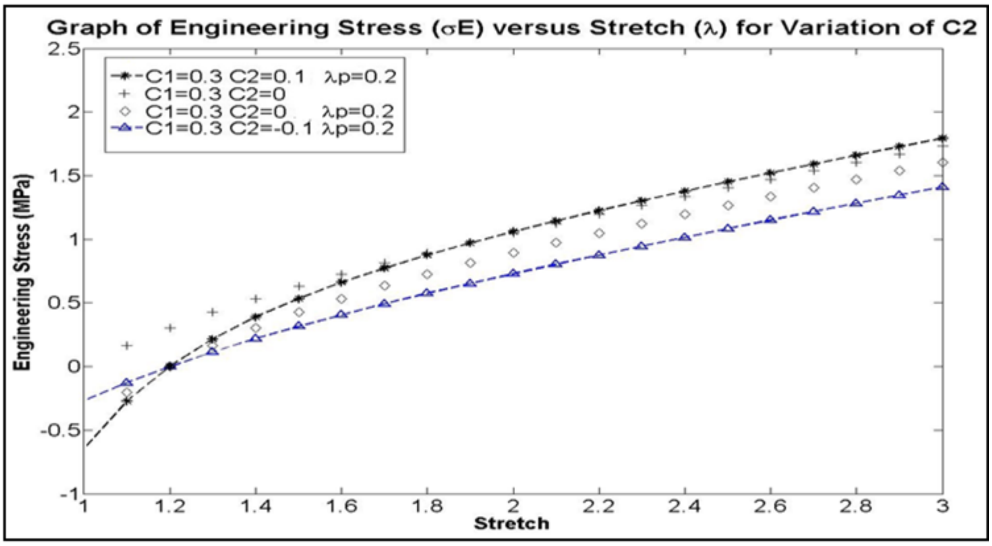

Figure 4. Variations in λ_p with fixed C1 and fixed C2

B. *Stage 2: Evaluation of Prestretch Influence of Mooney-Rivlin Material Model*

Fig. 1, Fig. 2 and Fig. 3 illustrate three different sets of Mooney-Rivlin model constants variations results. Graph of engineering stress, σ_E versus principal stretch, λ was used to represent the sensitivity of Equation (3) to the variation of the main constants in Mooney-Rivlin model (i.e. C_1, C_2 and λ_p). By referring to the known value of C_1 and C_2 by Shergold *et al.* [14] and λ_p by Evans and Holt [21], two reference graphs were plotted in each set. One is by using equation (2) which disregards the prestretch term and the other is by using equation (3) which includes the prestretch term.

Set1-Variation of C_1: The aim of Set 1 is to observe the effects of variations in C_1 value with fixed C_2 and λ_p value. Therefore, by remaining C_2 (0MPa) and λ_p (0.2) value at constant; according to Shergold *et al.* [14] and Evans and Holt [21], C_1 was varied from 0.2 to 0.4MPa (ΔC_1=0.2). Fig. 2 illustrates a stress-stretch diagram showing the sensitivity of the equation (3) to the variation of C_1 value. The horizontal axis represents stretch value while stress value appears on the vertical axis. Four stress-stretch curves were plotted including two reference graph. It is observed that, from the reference curve with and without prestretch terms, variation of C_1 produce quite a change in stresses value as the curve keep distancing each other at the end of the plot where large C_1 value indicates large engineering stress value. This appears to be that equation (3) is sensitive to the change of C_1 value.

Set2-Variation of C_2: Fig. 3 shows the results obtained from the variation of C_2 value from -0.1 to 0.1 (ΔC_2=0.2) by keeping C_1 (0.3MPa) and λ_p (0.2) value at constant; according to Shergold *et al.* [14] and Evans and Holt [21]. Again four stress-stretch curves were plotted including two reference graph mention earlier. The second set aimed to observe the effects of variations in C_2 value with fixed C_1 and λ_p value. It is observed that the curve produced for positive value of C_2 (0.1MPa) was quite closed to the reference curve (without prestretch term unlike the curve plot for negative value of C_2 (-0.1MPa). It is quite interesting to see the existence of a cross-sectional point of three curve plot at about 1.2 stretch value and 0MPa stress value. This discloses that a number of materials constants can produce a similar skin deformation. On top of that, equation (3) however seems to be less sensitive to the variation of C_2 value compared to Set 1 as small curve plot differences were produced with 0.2 unit difference in C_2 (ΔC_2=0.2).

Set3-Variation of λ_p: The value of λ_p was varied from 0.1 to 0.3 ($\Delta\lambda_p$ = 0.2) by C_1 (0.3 MPa) and C_2 (0MPa) remain constant; referring to Shergold *et al.* [12] and Evans and Holt [18]. As shown Fig. 4 with four stress-stretch curves including two reference graphs, all of the curves appear to be the same in trend. The third set aimed to observe the effects of variations in λ_p. It is shown that, large prestretch value induced less stress at constant C_1 and C_2 value. Therefore, this reveals that prestretch terms do influence in the determination of skin behaviour and properties.

Still, variation of C_1 and C_2 display significant influence to the stress-stretch curve despite from prestretch with the same unit difference of 0.2. This shows that equation (3) is very sensitive to the change of C_1 and C_2. All of the sets illustrate different stress-stretch curve plot, by mean material constants value influence its properties and behaviour. Nevertheless, the knowledge and findings gathered from this study is valuable and important to the determination of skin behaviour and properties as it consider the influence of skin prestretch.

IV. Conclusion

This study found that skin prestretch influence the determination of skin behaviour and properties. Based on Mooney-Rivlin material model, the objective of the study in investigating the influence of skin pre-stretch to the stress-stretch relations in defining skin behaviour and properties through a parametric study has been achieved successfully. Thus, the results obtained from this study could contribute to the study of hyperelastic constitutive model of skin in the future.

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